A METHOD OF INFERENCE AND DEFUZZIFICATION FUZZY INTERVAL

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ABSTRACT

In this work it is presented a fuzzy interval method of inference and defuzzification, that it is an extension of the traditional methods of inference and defuzzification, having as objective minimizes the mistakes of the specialist's imprecision, facilitating, therefore, the acquisition of knowledge, with the establishment of a margin of error through the use of intervals

KEY WORDS: Representation of the Approximate Reasoning, Fuzzy Systems, Fuzzy Inference, Fuzzy Defuzzification, Interval Theory, Fuzzy Interval Systems.

1. INTRODUCTION

The development of systems that map the human reasoning and that treat the imprecise knowledge it has been used with success by the fuzzy systems. But, in some situations, only this technology doesn't fully satisfy all the requirements of the problem. For instance, in systems that need to treat the data with a larger precision, it is difficult for the specialist to determine this imprecision degree through the pertinence values, been, this way, necessary another technology to aid the specialist and the knowledge engineer, facilitating the knowledge acquisition. For this kind of problems is indispensable the use of the intervals theory, or also that considers all the values supplied by the specialists.

In this way, this work comes to provide the use of intervals in the development of fuzzy systems. This addition comes to propitiate a larger quality in the data supplied by the specialists, in the sense that he can establish a margin of error in these data, improving the acquisition of the knowledge, because it's possible, for instance, to obtain several specialist's information and then to establish the interval with best represents the values supplied by them.

The fuzzy interval systems have been an object of study, during the last years, in several parts of the world, could be mentioned works like [1], [2], [3], [4], [5], [6], [7], [8], among others.

To soften the problem of the imprecision, this work proposes the use of a fuzzy interval method of inference and defuzzification, based on the use of the interval membership function.

For space subject, it will be taken in consideration that the reader already knows the fuzzy theory [9], [10], [11], [12], [13], [14] as well as the interval theory [15], [16], [17], [18]. However, it is important for a better understanding of this work that the continuity of functions theorem be placed to proceed. In the section 2, it will be made an introduction to the fuzzy interval theory, showing the concepts of interval membership function and some operations for fuzzy interval sets; the section 3 presents the fuzzy interval inference is and in the fourth section the fuzzy interval defuzzification is introduced, the conclusion is showed in the section 5 followed by the acknowledgments in the section 6.

1.1. CONTINUITY

The continuity notion is a notion that came from topologic spaces. The set of the real numbers, R, as well as RXR, have a well accepted topology in the mathematical world, and therefore a continuity notion. That continuity notion coincides with the geometric vision of functions continuity in the Cartesian plan. How to IR can be faced as a subset of RXR, it is reasonable to say that an interval function is continuous if, and only if, it is continuous in the IR topology induced by RXR. A classic result is that a function f: R to RXR is continuous if exists two functions f_1: R → R and f_2: R → R continuous such that $f = f_1 X f_2$ [19]. A natural extension of that result to IR is the following:

Theorem 1: Let f: R → IR. f is continuous if, and only if, exist the continuous functions f_i: R → R e f_s: R → R such that, ∀x∈R, f(x)=[f_i(x), f_s(x)]. Therefore $f_i(x) \le f_s(x)$ for each x ∈ R.

2. THEORY FUZZY INTERVAL

Some researchers are working with the notion of "Interval Fuzzy Valued Sets (IVFS)", like Turksen that proposes that theory initially in 1986 [20]; Rocha in [1], [2], [3] that proposes, based on Turksen the "Evidence Sets", in [5] the notion of intervals is shown as equal of fuzzy values, finally in [7], [8], that describes an integration of the interval theory to the fuzzy systems.

That process involves several stages, from the definition of the operations and properties passing to the generation of the theory, until the construction of the function of interval membership. Besides, it should be, also, built inference methods and defuzzification to close the specification of the system fuzzy interval. To proceed it will be shown the definition of function of interval membership and an example of fuzzy interval operators.

2.1. MEMBERSHIP FUNCTION

The construction of the interval membership function is made using intervals in the fuzzy membership function. This way, instead of treating the membership value as a real number in its image, the defined membership function for the fuzzy interval set will work with an interval of real numbers in the interval [0, 1]. In other words, a sub-interval of the interval [0, 1] in its image. This way, the fuzzy interval set

$$A = \{(x, \varphi_A(x))\},$$

where x ∈ U and $\varphi_A(x) \in I[0,1]$, with $I[0, 1] = \{ [a, b] \in IR \ / \ 0 \le a \le b \le 1\}$. U is the universe in discussion, $\varphi_A(x)$ is the interval membership degree for the fuzzy interval set A.

The interval membership function is defined in:

φ_A: U → I [0,1]

where U is the universe set, that will be the set of the real numbers in this work, because most of the practical cases works with the numeric universe.

The interval membership degree for a fuzzy interval set A, is given by $\varphi_A(x)$, satisfying the following theorem 1, this is, there is functions φ_{Ai}, φ_{As} : R → [0, 1] continuous, such that ∀x ∈R, $\varphi_A(x) = [\varphi_{Ai}(x), \varphi_{As}(x)]$, therefore $\varphi_{Ai}(x) \le \varphi_{As}(x)$, where φ_{Ai} is called function of inferior limit and φ_{As} of function of superior limit.

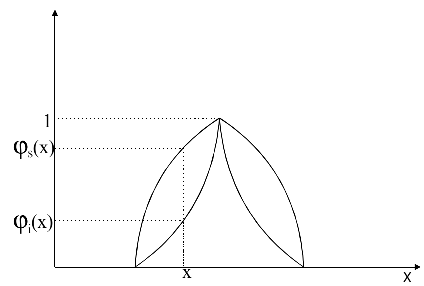

Figure 1. **Fuzzy Interval Set**

The interval membership function φ is given by the juxtaposition of the functions of inferior limit φ_{Ai} and superior limit φ_{As}. The figure 1 shows a fuzzy interval set. The degree of interval membership $\varphi_A(x) = [\varphi_{Ai}(x), \varphi_{As}(x)]$. It is important to point out that, not always the limiting functions (φ_{Ai} and φ_{As}) possess the uniform behavior as the one in the illustration 1.

2.2. OPERATIONS AND PROPERTIES

In the development of the fuzzy interval theory the relationships between the fuzzy operators and the interval operators should be established. All the fuzzy operators can be naturally extended to intervals [21]. For instance, the defined basic operations for fuzzy sets:

Union: A ou B = {(x, max ($\mu_A(x)$; $\mu_B(x)$))}

Intersection: A ∩ B = {(x, min (($\mu_A(x)$; $\mu_B(x)$))}

Complement: ¬A = {(x, $\mu_{\neg A}(x)$)| $\mu_{\neg A}(x) = 1 - \mu_A(x)$}

where x ∈ U (universe in discussion), and μ_A is the membership degree for the fuzzy set A, they can be extended to fuzzy interval sets.

They can be defined for fuzzy interval sets. Reminding the interval definition:

$X = [a, b] = \{ x \in R \ / \ a \le x \le b\}$.

Defining the operations MIN and MAX, the minimum and the maximum for intervals, as:

$MAX([a_1, a_2]; [b_1,b_2]) = [max(a_1,b_1),max(a_2,b_2)]$

$MIN([a_1, a_2]; [b_1,b_2]) = [min(a_1,b_1),min(a_2,b_2)]$

Let A and B fuzzy interval sets, with $\varphi_A(x) = [a_1, a_2]$ e $\varphi_B(x)=[b_1,b_2]$.

Therefore, the same operations can be defined for fuzzy interval sets as:

Union: $= \{(x, MAX(\varphi_A(x),\varphi_B(x)))\} = \{(x, [max(a_1,b_1), max(a_2,b_2)])\}$

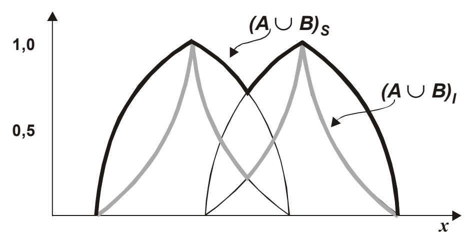

Figure 2 - Fuzzy Interval Membership Function - Union

Intersection: $= \{(x, MIN(\varphi_A(x),\varphi_B(x)))\} = \{(x,[min(a_1,b_1), min(a_2,b_2)])\}$

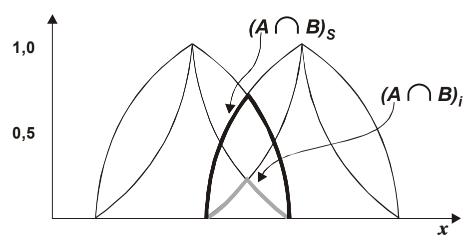

Figure 3 - Fuzzy Interval Membership Function - Intersection

Complement: $= \{(x, 1 - \varphi_A(x))\} = \{(x, [1-a_2, 1-a_1])\}$

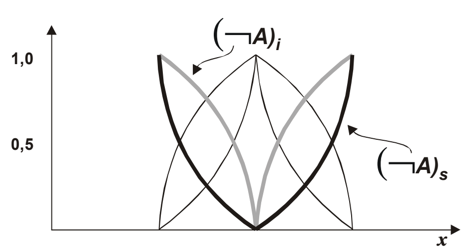

Figure 4 - Fuzzy Interval Membership Function - Complement

The fuzzy proprieties also can be naturally extended to intervals as the *commutative*, *associative*, *distributive* proprieties.

For instance, consider A and B fuzzy interval sets, the commutative property for union, intersection is valid.

$A \cup B = B \cup A$

$A \cap B= B \cap A$

3. FUZZY INTERVAL INFERENCE

In the fuzzification of the concepts [9], [10], [22] a membership function is built, could exist different membership functions for the same concept, in agreement with the specialist's subjectivity [9], [13].

In that way the choice of a certain membership function for a concept generates an uncertainty, a mistake, with relationship to the membership values. For the degrees of 0, or totally false, and 1, or totally true, can be said that this mistake is not important. However when these values are going close to the center of the interval [0, 1], those uncertainties tend to increase more and more [8], [11].

To reduce the uncertainty about the membership degree, can be defined two other functions that will delimit and reduce the specialist's mistake. This way, the membership degree can be as display the figure 1.

The fuzzy interval inference is given through use generalized modus ponens inference [12], [23]:

$B' = A' \otimes \Re(x, y) = A' \otimes (A \rightarrow B)$

where A, A' B and B' are interval fuzzy sets, x and y are fuzzy variables, $\Re(x,y)$ is the interval fuzzy binary relationship of implication, $\Re(x,y) = \{((x,y), \varphi\Re (x,y))\}$, and $\otimes$ it is the interval fuzzy composition operator.

In the case of the min-max interval fuzzy inference they will be defined the composition operators and of min-max interval implication.

The interval min-max composition for the degree of interval membership of two fuzzy interval relationships $\Re_1$ e $\Re_2$ is given by:

$$\varphi_{\Re 1 \circledcirc \Re 2}(x,z) = MIN_y\,(MAX(\varphi_{\Re 1}(x,y),\ \varphi_{\Re 2}(y,z)))$$

It can be defined in terms of functions of intervals membership as:

$$\varphi_{B'}(y) = MIN_x\ (MAX(\varphi_{A'}(x),\ \varphi_{\Re}\ (x,\ y)))$$

where $\varphi_{B'}(y)$ is the interval membership function of B', $\varphi_{A'}(x)$ is the interval membership function of A', and $\varphi_{\Re}$ (x, y) is the membership function of the relationship of interval implication.

The relationship of interval min-max implication is defined as:

$$\varphi_{\Re}\ (x,y) = MIN(MAX(\varphi_A(x),\ \varphi_B(y)),\ (1-\varphi_A(x)))$$

For instance, using the inference min-max, and applying the concepts of the fuzzy interval inference in the following fuzzy interval set A (figure 5) and B (figure 6). Observe that the degree of interval membership is an interval, i.e. φ_A (4) = [0.4, 0.6]

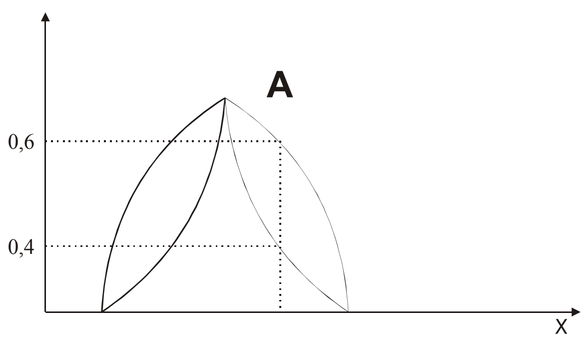

Figure 5 - Fuzzy Interval Set A

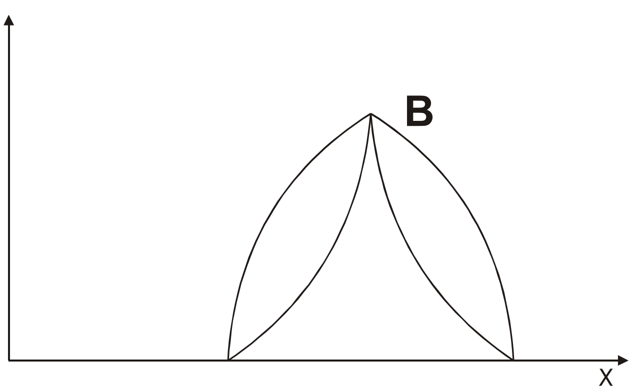

Figure 6 - Fuzzy Interval Set B

Consider the following situation now: let be A' a fuzzy interval set given as input, where the interval membership degree for $\varphi_{A'}(4)$ = [1, 1] and for the A set, the membership interval degree $\varphi_A(4)$ = [0.4, 0.6], and a generic rule If x is A then y is B. The fuzzy area solution generated, in this case, will be two areas: an area for the lower limit function and other for the upper limit function. For the lower limit function it is had that the min {1, 0.4} it will cut the function of the lower limit of the interval set B. And for the upper limit function the min{1, 0.6} it will cut the function of the upper limit of the interval set B, represented respectively in the figures 7 and 8.

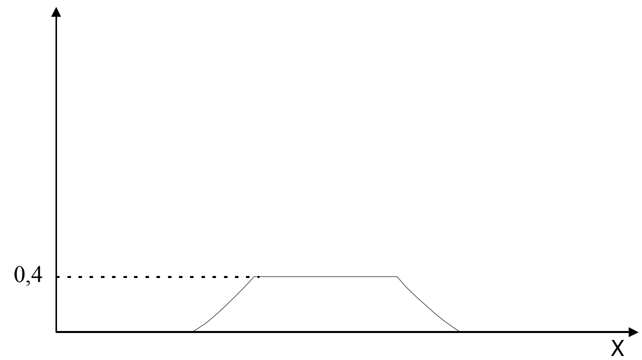

Figure 7 - Inferior Solution Set

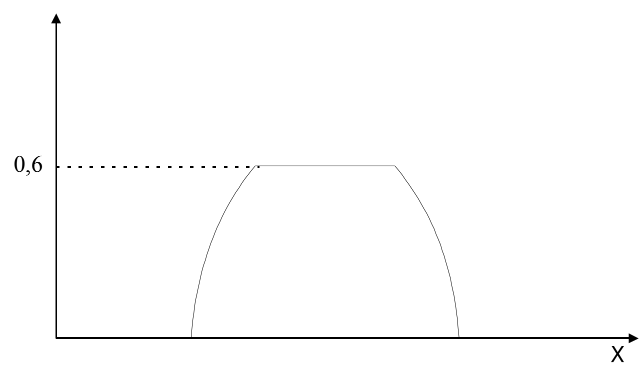

Figure 8 – Superior Solution Set

4. INTERVAL DEFUZZIFICATION

The process of interval defuzzification is obtained through to defuzzification of the lower (d_i) and upper (d_s) solution sets, through any of the methods of fuzzy defuzzification. The result is an interval formed by the value of the defuzzification of the lower (di) and upper (ds) function:

DI = [min(d_i, d_s), max(d_i, d_s)].

To calculate a single value as solution for to interval defuzzification the medium point of the found interval is extracted:

d = (d_i + d_s) / 2

For the defuzzification of the previous example the defuzzification method to be used will be the Maxplateau method that is given for: [9], [10], [12].

$u^* = \sum u_m / M$

where u_m is the m^{th} element in the universe in discussion, the membership degree $\mu(u_m)$ is in the maximum value of the membership function, and M is the total number of elements.

The interval version of this defuzzification method is as follows:

$d_i = \sum u_{mi} / M$

$d_s = \sum u_{ns} / N$

where u_{mi} and u_{ns}, are m^{th} and n^{th} elements in the universe in discussion with a maximum pertinence degree in solution sets inferior and superior, respectively. Thus, **DI** and **d** can be obtained as in the equations above.

As the boundary functions are proportional in this case, the maximum values of the functions are approximately:

$d_i = (6.7 + 9.3) /2$

$d_i = 8$

and

$d_s = (6.3 + 9.7)/2$

$d_s = 8$

then,

DI = [8, 8]

and

d = (8 + 8) /2

The output for that problem, or the defuzzification, receive an approximately value equal to 8, as show the figures 9 and 10.

Notice that in the previous example, uniform boundary functions were used, get the same answer for the lower (d_i) and upper (d_s) values of the defuzzification. If other defuzzification method were used as the centroid, for instance, d_i would continue equal to d_s because the boundary functions are symmetrical. However it is worth to point out that, not always that happens, because when the boundary functions are variables, this is not proportional, the values of the defuzzification certainly will be different.

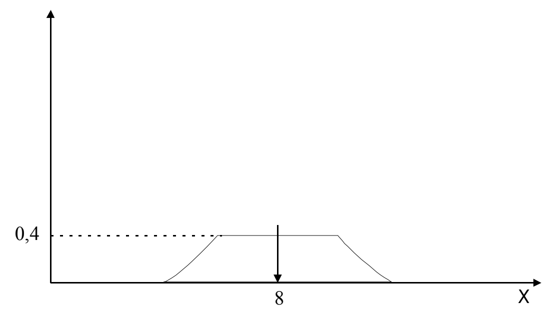

Figure 9 - Maxplateau Defuzzification on Inferior Solution

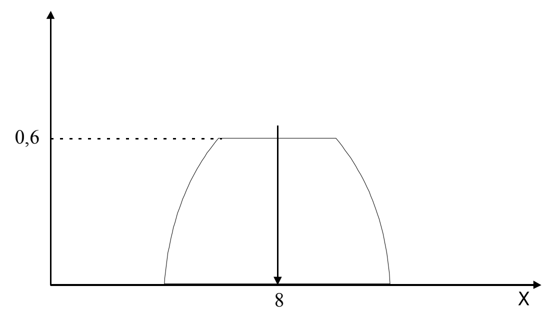

Figure 10 - Maxplateau Defuzzification on the Superior Solution

5. CONCLUSION

The integration of intervals to fuzzy systems is a subject that is being enough studied in the scientific community, but isn't totally formed yet.

This is a work of theoretical investigation, doesn't having any implementation result yet. However, this is the next step, after the fuzzy interval theory be formulated.

The largest contribution of this work is in improving the process of acquisition of the specialist's knowledge, in the sense that the specialist doesn't need to worry so much with the precision of the membership degrees supplied.

The method used in the inference is just a proposal among many that are appearing with the study of the fuzzy interval theory, and other methods should appear in the next works. It's possible to obtain variations using different implication operators, as well as in the form of establishing the boundary functions of the fuzzy interval sets.

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