A note on solving fuzzy differential equations

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Abstract. In this article we present some remark on solving fuzzy differential equations. Particularly, we show that a fuzzy differential equation may not always be replaced by an equivalent system of parametric differential equations. We given a class of fuzzy differential equations where this equivalence is valid and any classical numerical method for differential equations can be adapted to obtain a fuzzy solution.

Keywords: Fuzzy differential equations, solving fuzzy differential equations, extension principle

1 Introduction

We consider the fuzzy differential equation

\[ \begin{aligned}
X'(t) &= F(t, X) \\
X(0) &= X_0,
\end{aligned} \]

where \( F : [0, T] \times F_C \to F_C \) is a continuous fuzzy function, \( X_0 \in F_C \), \( F_C \) is the space of all fuzzy intervals and \( X' \) denotes the strongly generalized derivative of the fuzzy-valued function \( X \).

It is well-known that the fuzzy differential equation (1) may be replaced by an equivalent system of parametric differential equations \([1–3, 6, 7, 11, 12, 15–17, 19, 23–26]\). Thus, for we obtain a solution of the fuzzy differential equation (1), when it exists, we have to solve the system of parametric differential equations. This fact has been well used for implement, adapt classical numerical methods for differential equation to fuzzy context, for instance see \([1–3, 11, 12, 15–17, 19, 23–26]\).

However, the fuzzy differential equation (1) may not always be replaced by an equivalent system of parametric differential equations in the form that it is considered in some papers, as we are going to show. But if we consider that \( F \) is generated by applying the Zadeh’s extension principle to a real function of variable real then we have this equivalence.

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