Actions of Automorphisms on Some Classes of Fuzzy Bi-implications

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Abstract. In a previous paper we have studied two classes of fuzzy bi-implications based on t-norms and r-implications, and shown that they are increasingly weaker subclasses of the Fodor-Roubens bi-implication. Now we prove that each of these three classes of bi-implications are closed under automorphisms.

Keywords: fuzzy bi-implications, automorphisms, fuzzy operators

1 Introduction

In [9] we studied the relation between the more well-known definition proposed by Fodor and Roubens and other appealing definitions, old or new, of fuzzy operators that extend the interpretation of the classical bi-implication.

On the other hand, automorphisms, i.e., isomorphisms between the same lattices, with the composition form a group [22, 3]. Automorphisms had played an interesting role in fuzzy connectives, because when a class of fuzzy connectives is closed under automorphisms the action of the group of automorphisms establishes an equivalence relation between the connectives and therefore determine a partition among these connectives. These partitions, in some case have characterized important subclasses of fuzzy connective. For example, the class of strict t-norms is the equivalence class of the product t-norm [16], the class of nilpotent t-norms agree with the equivalence class of the Lukasiewicz t-norm [16], the class of strong negations is the same that the equivalence class of the standard negation [26], and the class of implications which are both strong and residual is the equivalence class of the Lukasiewicz implication [1]. So it is reasonable to study the action of automorphisms on fuzzy bi-implications. In this paper we prove that each of the three classes of fuzzy bi-implications studied in [9] are closed under automorphisms.

2 Fuzzy extension of conjunction and implication

Definition 21 A triangular norm (in short t-norm) is a binary operator $T$ on the unit interval $[0, 1]$ that: on $\{0, 1\}$ behaves as classical conjunction, is