Abstract. We apply the concepts of fuzzy sets to lie algebras in order to introduce and to study the notions of solvable and nilpotent fuzzy radicals. We present conditions to prove the existence and uniqueness of such radicals and investigate some of their properties.

Keywords: Fuzzy Lie algebra, Solvable fuzzy radical, Nilpotent fuzzy radical.

1 Introduction

Radical of algebras was a concept that first arose in the classical structure theory of finite-dimensional algebras at the beginning of the 20th century. Initially the radical was taken to be the largest nilpotent ideal of a finite-dimensional associative algebra. Algebras with zero radical (called semi-simple) have obtained a fairly complete description in the classical theory: Any semi-simple finite-dimensional associative algebra is a direct sum of simple matrix algebras over suitable fields. At the same time it turned out that the radical, as well as the largest solvable or largest nilpotent ideal, could be defined in the class of finite-dimensional Lie algebras. Here, as in the associative case, semi-simple Lie algebras turned out to be direct sums of simple algebras of some special form [2].

The notion of fuzzy sets was introduced by L. A. Zadeh [5] and the notions of fuzzy ideals and fuzzy subalgebras of Lie algebras over a field were first introduced by Yehia in [4]. Many mathematicians have been involved in extending the concepts and results of Lie algebras to the fuzzy sets. The aim of this paper is to generalize the concept of solvable and nilpotent radical of Lie algebras to the notion of solvable and nilpotent fuzzy radical of a fuzzy algebra of Lie algebras, respectively, and investigate some of their properties.

2 Fuzzy sets

In this section, we present the basic concepts on fuzzy sets which will be used throughout this work. A notion is introduced and a result is proved for guiding the construction of the main theorems of this work.