Sparsely Connected Semilattice Associative Memories on Certain \(\mathbb{L}\)-Fuzzy Sets*

Peter Sussner\(^1\) and Marcos Eduardo Valle\(^2\)

\(^1\) Department of Applied Mathematics, University of Campinas (Unicamp). CEP 13081-970, Campinas – SP, Brazil. E-mail: sussner@ime.unicamp.br

\(^2\) Department of Mathematics, University of Londrina (UEL). CEP 86051-990, Londrina – PR, Brazil. E-mail: valle@uel.br

Abstract. In mathematical morphology (MM), images are viewed as \(\mathbb{L}\)-fuzzy sets, where the symbol \(\mathbb{L}\) stands for a complete lattice. In particular, fuzzy MM arises by considering \(\mathbb{L} = [0, 1]\). Mathematical morphology provides the theoretical basis for certain lattice computing models called morphological neural networks (MNNs) including morphological associative memories (MAMs) that are the focus of this paper. A closer look reveals that a MAM yields a mapping from one class of \(\mathbb{L}\)-fuzzy sets (not necessarily representing images) to another. Note that for a complete lattice \(\mathbb{L}\) the class of \(\mathbb{L}\)-fuzzy sets is also a complete lattice. Apart from some excellent characteristics, classical MAM models also exhibit a major disadvantage, namely a very limited noise tolerance, due to the duality principle which represents an intrinsic property of MM on complete lattices. Therefore, we resort to the recent theory of MM on complete inf-semilattices as a framework for a sparsely connected associative memory that is capable of dealing with the computational requirements for storing multi-valued and large-scale patterns. In this context, the patterns are given by \(\mathbb{L}\)-fuzzy sets where \(\mathbb{L}\) is a certain type of a conditionally complete lattice that can be used to construct an inf-semilattice. This approach yields different error correction capabilities than classical MAM models because MM on complete inf-semilattices is endowed with self-dual operators. We finish with some experimental results concerning the problem of storing and reconstructing gray-scale as well as color images.

Keywords: \(\mathbb{L}\)-fuzzy set, mathematical morphology, complete lattice, conditionally complete lattice-ordered group, inf-semilattice, associative memory, gray-scale and color images.

1 Introduction

Generalizing the concept of a fuzzy set, Goguen introduced \(\mathbb{L}\)-fuzzy sets in his seminal paper of 1967\(^1\). Recall that an \(\mathbb{L}\)-fuzzy set is given by a mapping from a universe \(X\) to a set \(\mathbb{L}\). In this setting, \(\mathbb{L}\) has a mathematical structure that is at least a partially

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