Fuzzy Modal Logics of Confluence

Adriano Alves Dodó
João Marcos

Group for Logic, Language, Information, Theory and Applications (LoLITA)
Department of Informatics and Applied Mathematics (DIMAp)
UFRN, Brazil
adrianododo@ppgsc.ufrn.br
jmarcos@dimap.ufrn.br

Abstract

In this paper we explore fuzzy semantics for a wide class of normal modal systems enriched with multiple instances of the axiom of confluence and prove a general completeness result for such systems.

Keywords: fuzzy logics, modal logics, confluence

1 Introduction

In [3, 1], the authors study models for a certain kind of fuzzy modal logics and prove weak completeness results for a couple of modal extensions of classic-like fuzzy models of some traditional normal modal systems, viz. \( K, T, D, B, S4 \) and \( S5 \). Here we shall follow a similar thread to prove completeness results for a much more inclusive class of fuzzy normal modal systems which contain instances of the axiom of confluence, as the systems \( K + G^k,l,m,n \) obviously encompass the above traditional systems, and much else. Indeed, one may observe that the characteristic modal axioms \( pTq \), \( pDq \), \( pBq \), \( p4q \) and \( p5q \) are but particular instances of \( pG_{k,l,m,n}q \) where \( xk,l,m,ny \) are \( x0,0,1,0y \), \( x0,1,0,1y \), \( x0,1,1,0y \), \( x1,1,0,1y \), respectively.

The so-called Geach axiom \( pG_{1,1,1,1}q \) is well-known to characterize, in terms of the associated notion of accessibility (and its inverse) in the corresponding Kripke frames, the diamond property, namely: if \( y \leftarrow x \rightarrow z \), then there is some \( w \) such that \( y \rightarrow w \rightarrow z \). As noted in [5], where \( \leftarrow \rightarrow \) denotes an \( i \)-long sequence of \( \leftarrow \rightarrow \) transitions (and similarly for \( \leftarrow \rightarrow \) transitions), the natural generalization of the diamond property is the following \( \langle k, l, m, n \rangle \)-confluence property: if \( y \leftarrow k x \rightarrow l z \), then there is some \( w \) such that \( y \rightarrow m w \rightarrow n z \). From the logical viewpoint, a general completeness proof based directly on the axiom of confluence, thus, is attractive in having the potential to subsume a denumerable number of particular instances of \( G^{k,l,m,n} \).

At any rate, it should be noted that the confluence property has importance on its own. In abstract rewriting systems, for instance, one deals with frames in which accessibility characterizes some appropriate notion of reduction. There, confluence is used together with termination to guarantee convergence of reductions, which on its turn guarantee the existence of normal forms and has applications on the design of decision procedures. Strong normalization, in particular, is a much desirable property of lambda calculi, and is a property guaranteed by theorems of confluence à la Church-Rosser, with applications to programming language theory. The availability of modal logics of confluence, and in fact of fuzzy versions of such logics, allows one to expect to have a local perspective on rewrite systems and on