Fuzzy Differential Equations with Arithmetic and Derivative via Zadeh’s Extension

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Abstract. We adopt the derivative for fuzzy functions obtained via Zadeh’s extension of the classical derivative operator [1] and implement it in dynamical systems. Particularly, we explicit solutions to fuzzy initial value problems (FIVPs) that preserve the main properties and characteristics of functions of the base space, as periodicity and stability. This is a known feature of fuzzy differential inclusions (FDIs). However, unlike solving inclusions, we study a theory for fuzzy differential equations (FDEs). Some examples are provided to illustrate the theory and the solutions to FIVP are compared with those from other approaches.

Keywords: Fuzzy Derivative, Fuzzy Differential Equation, Zadeh’s Extension, Fuzzy Arithmetic

1 Introduction

Zadeh’s extension is considered a very powerful technique in fuzzy sets theory. It is a case of united extension, that is, it extends functions whose arguments are points to functions whose arguments are sets. Denoting by \( \mathcal{F}(E) \) the space of fuzzy subsets of a topological space \( E \), Zadeh’s extension, in particular, returns functions over the space \( \mathcal{F}(E) \), given that these functions were originally defined over the space \( E \). What is interesting is that such extended functions inherit the main properties and characteristics of the original function.

In fuzzy dynamical systems, many approaches make use of Zadeh’s extension. One example is the approach first proposed by Oberguggenberger [2] and later studied by Mizukoshi et al. [3], consists of solving an initial value problem (IVP) and extending the solution according to fuzzy parameters. It has been proved [3] that under certain conditions, these solutions have the same attainable sets of those obtained via FDIs.

It is also a common practice to define the field of a FIVP as Zadeh’s extension of some classical function. Chalco-Cano et al. [4] claim that this is an optimal interpretation and argue for the resulting arithmetic.

The usual fuzzy interval arithmetic (Moore’s interval arithmetic) is very simple to compute, since it operates only with interval endpoints. However, it leads