ART for helical cone-beam CT reconstruction

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Abstract

We report on our first results on the use of Algebraic Reconstruction Techniques (ART) on helical cone-beam Computerized Tomography (CT) data. Two variants of ART have been implemented: a standard one which considers a single ray in an iterative step and a block version which groups several cone-beam projections in calculating an iterative update. Both seem to produce high-quality reconstructions, although the number of cycles through the data to achieve those (between 15 and 20), while not huge, is larger than the number of cycles through the data needed for reconstructing volumes from data acquired from different modalities (1 iteration for PET data and 1 to 4 iterations for EM data). The reason for that maybe due to the uneven coverage of points by the data collection geometry, resulting in a slower rate of convergence.

I. Introduction

Algorithms for image reconstruction from projections form the foundations of modern methods of tomographic imaging in radiology, such as helical cone-beam X-ray computerized tomography (CT). Helical cone-beam CT is an image modality in which the cone-beam data acquisition is performed with a helical motion of the X-ray source-detector relative to the patient. The value of the helix pitch determines the speed of data acquisition, the bigger the pitch value, the faster the acquisition is. In [1] we showed that it is possible to obtain high-quality reconstructions from helical cone-beam CT data using ART (Algebraic Reconstruction Technique) even when applied to data acquired when using a considerably big pitch value.

An image modeling tool, which was described in a general context in [2, 3] and utilized in image reconstruction algorithms in [4, 5], is the representation of images and volumes using *blobs*, which are radially symmetric bell-shaped functions whose value at a distance r from the origin is

$$b_{m,a,\alpha}(r) = \frac{1}{I_m(\alpha)} \left[\sqrt{1-(r/a)^2}\right]^m I_m\left[\alpha\sqrt{1-(r/a)^2}\right], \tag{1}$$

for $0 \leq r \leq a$ and is zero for $r > a$. In this equation I_m denotes the modified Bessel function of order m, a is the radius of the support of the blob and α is a parameter controlling the blob shape. A volume is represented as a superposition of N scaled and shifted versions of the same blob; i.e., as

$$\overline{f}(x,y,z) = \sum_{j=1}^{N} c_j b_{m,a,\alpha}\left(\sqrt{(x-x_j)^2+(y-y_j)^2+(z-z_j)^2}\right), \tag{2}$$

where $\{(x_j, y_j, z_j)\}_{j=1}^N$ is the set of grid points in the three-dimensional (3D) Euclidean space to which the blob centers are shifted. Once we have chosen these grid points and the specific values of m, a and α, the volume is determined by the finite set $\{c_j\}_{j=1}^N$ of real coefficients; the task of the reconstruction algorithm in this context is to estimate this set of coefficients from the projection data.

The aim of [4, 5] was to study the choices of the grid points and of the parameters m, a and α, combined with implementation of the algorithm to estimate the coefficients, from the point of view of obtaining high-quality reconstructions in a reasonable time.

II. Helical cone-beam reconstruction using ART

It has been pointed out in [6] that applying the simplest form of ART to cone-beam projection data acquired on a circular trajectory can result in substandard reconstructions, and it has been suggested that a certain alteration of ART leads to improvement. However, besides an illustration of its performance, no properties (such as limiting convergence) of the algorithm have been given. We still need a mathematically rigorous extension of the currently available theory of optimization procedures to include acceptable solutions of problems arising from cone-beam data collection. We discuss this phenomenon in the context of reconstruction using ART with blobs from helical cone-beam data collected according to the geometry of [7]. In [1] we showed that ART can indeed produce high-quality results when applied to helical cone-beam data. In this paper we will concentrate on how to improve the convergence rate of the reconstruction algorithm by making use of a block-ART algorithm.

A. Standard ART

For this discussion we adopt the notation of [8], because it is natural both for the assumed data collection and for the mathematics that follows. We let I denote the number of times the X-ray source is pulsed as it travels its helical path multiplied by the number of lines for which the attenuation line integrals are estimated in the cone-beam for a single pulse. Thus I is the total number of measurements and we use Y to denote the (column) vector of the individual measurements y_i, for $1 \leq i \leq I$. We let N denote the number of grid points at which blobs are centered; our desire is to estimate the coefficients $\{c_j\}_{j=1}^{N}$ and thereby define a volume using (2). For $1 \leq i \leq I$, we let a_{ij} be the integral of the values in the jth blob along the line of the ith measurement (note that these a_{ij} can be calculated analytically for the actual lines along which the data are collected) and we denote by A the matrix whose ijth entry is a_{ij}. Then, using c to denote the (column) vector whose jth component is c_j, this vector must satisfy the system of approximate equalities:

$$Ac \approx Y. \tag{3}$$

In the notation of [8] the traditional ART procedure for finding a solution of (3) is given by the iterations:

$$\begin{aligned} & c^{(0)} \quad \text{is arbitrary}, \\ & c_j^{(n+1)} = c_j^{(n)} + \omega^{(n)} \frac{y_i - \sum_{k=1}^{N} a_{ik} c_k^{(n)}}{\sum_{k=1}^{N} a_{ik}^2} a_{ij}, \\ & \qquad \text{for } 1 \leq j \leq N, \\ & n = 0, 1, \ldots, \quad i = n \bmod I + 1, \end{aligned} \tag{4}$$

where $\omega^{(n)}$ is a relaxation parameter. While this procedure has a mathematically well-defined limiting behavior (see, e.g., Theorem 1.1 of [8]), in practice we desire to stop the iterations early for reasons of computational costs. We have found that for the essentially parallel-beam data collection modes of fully 3D PET [9], Fourier rebinned PET [10] and Transmission Electron Microscopy [11], one cycle through the data (i.e., $n = I$) is sufficient to provide us with high-quality reconstructions. However, our preliminary experiments indicate that this does not happen with helical cone-beam data.

We conjecture that the reason for this is the following. Let us associate with the jth blob the value

$$s_j = \sum_{i=1}^{I} a_{ij}, \text{ for } 1 \leq j \leq N. \tag{5}$$

For the parallel mode of data collection the values of s_j are nearly the same for all the blobs. However, this is not the case for cone-beam data. If we use the data collection geometry of [7], the blob coefficients closer to the helical source trajectory will have higher s_j values than the blob coefficients on the opposite side of the trajectory and, as can be seen in (4), this results in some blob coefficients being updated more frequently than others, making it harder for the iterative algorithm to converge to an acceptable solution.

B. Block-ART

It is natural to consider instead of the row-action algorithmic scheme (4) its block-iterative version, in which all the measurements taken by a number of pulses of the X-ray source form a block. A powerful theory is developed for this in [8]. Let M be the number of blocks, Y_i be the L-dimensional vector of those measurements which form the ith block and let A_i be the corresponding submatrix of A (we assume that each block has the same number of measurements). Theorem 1.3 of [8] states that the following block-iterative algorithm has good convergence properties:

$$\begin{aligned} & c^{(0)} \quad \text{is arbitrary}, \\ & c^{(n+1)} = c^{(n)} + A_i^T \Sigma^{(n)} \left(Y_i - A_i c^{(n)} \right), \\ & n = 0, 1, \ldots, \quad i = n \bmod M + 1, \end{aligned} \tag{6}$$

where $\Sigma^{(n)}$ is an $L \times L$ relaxation matrix. This theory covers even fully-simultaneous algorithmic schemes (just put all the measurements into a single block). There are also generalizations of the theory which allow the block sizes and the measurement-allocation-to-blocks to change as the iterations proceed.

A variation on such a block-ART algorithm is to perform component-dependent weighting in the update of blob coefficients. The essence of this approach is to introduce in (6) a second $(N \times N)$ relaxation matrix $\Delta^{(n)}$ in front of the A_i^T. Then we need to answer the following: For what simple (in the sense of computationally easily implementable) pairs of relaxation matrices $\Sigma^{(n)}$ and $\Delta^{(n)}$ can we simultaneously obtain desirable limiting convergence behavior and good practical performance by the early iterates. Examples of the $\Delta^{(n)}$ to be studied are the diagonal matrix whose jth entry is the reciprocal of the s_j of (5) or, alternatively, the reciprocal of a similar sum taken over only those measurements i which are in the block used in the particular iterative step. A recently proposed simultaneous reconstruction algorithm which uses j-dependent weighting appears in [12], where it is shown that a certain choice of such weighting leads to substantial acceleration of the algorithm's initial convergence.

Here we define the weights to be used in the updates based on the following idea. Suppose that we have taken the projection data of an object for which all the blob coefficients c_j are 1. Then, it appears desirable to have a uniform assignment of the blob coefficients after a single step of a modified version of (6), assuming that the initial assignment of the blob coefficients is zero. Assuming that the $\Sigma^{(n)}$ is the identity matrix, we can achieve this aim by choosing $\Delta^{(n)}$ to be a diagonal matrix whose jth entry is inversely proportional to the sum over all lines in the block of the line integral through the jth blob multiplied by the sum of the line integrals through all the blobs. The mathematical expression for this is

$$\sum_{l=1}^{L} \left(a_{[(i-1)L+l]j} \sum_{k=1}^{N} a_{[(i-1)L+l]k} \right). \tag{7}$$

In order for this to work we have to ensure that the value of (7) is not zero. This is likely to demand the forming of blocks which correspond to more than one pulse of the X-ray source, since the rays forming a block should intersect all blobs in the reconstruction region.

III. Results

Both ART (4) and the block-ART (described by (6) and (7)) algorithms were used to reconstruct a modified 3D Shepp-Logan phantom [13] in which the values range from 0.00 to 2.00, using data collected from two helix turns, with 300 projections taken per turn and 64 rows and 128 channels per projection (i.e. $I = 2 \times 300 \times 64 \times 128 =$4,915,200). The cone and fan angles of the cone-beam were 9.46° and 21.00°, respectively. The reconstructed volumes consisted of a $95 \times 95 \times 191$ blob coefficients array (2) organized on a bcc grid (see [4]) that was interpolated to a cubic grid with $128 \times 128 \times 128$ voxels. Figure 1 shows a (x,z)-slice of the volume reconstructed using the standard ART algorithm (a) and the block-ART algorithm (b) and a (y,z)-slice using the the standard ART algorithm (c) and the block-ART algorithm (d). The grayscale window used to show the slices was [1.00,1.03]. Both algorithms were executed for 17 cycles using $\omega^{(n)} = 0.01$ for the standard ART algorithm and 0.1 as a relaxation parameter (multiplying the identity matrix $\Sigma^{(n)}$) for the block-ART algorithm. For the block-ART algorithm, the measured data was grouped into 75 blocks formed by 8 cone-beams each. As one can see, the visual quality of these reconstructions is similar, although the blob-ART reconstruction seems to produce a more uniform backgroung inside the skull of the phantom. The time needed for both reconstructions is similar since the block-ART algorithm only carries a small overhead for computing the weights for each particular (blob,block) pair. (This overhead can be eliminated by precomputation and storage of the weights.)

IV. Discussion

We presented here our first results on the use of ART techniques for the reconstruction of helical cone-beam CT data. Our future work will concentrate on how to optimize the algorithms, by both speeding up the execution of a single cycle and improving the rate of convergence of the algorithms, and the evaluation of these algotihms and other block-ART variants.

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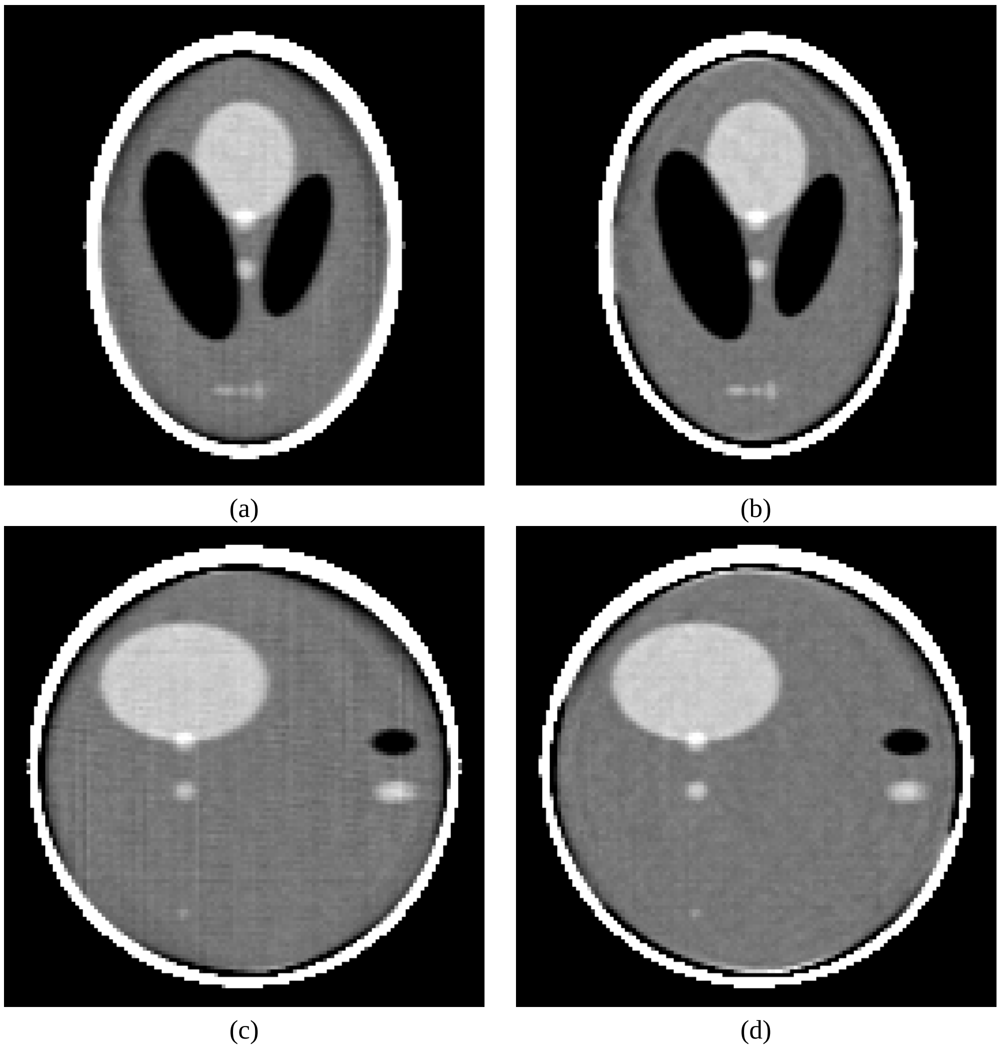

Figure 1: Slices of reconstructed Shepp-Logan phantom using the standard ART algorithm (a) and (c), and the block-ART algorithm (b) and (d), showed using the gray-scale window settings of [1.00,1.03] (images (a) and (b) show a (x,z)-slice while images (c) and (d) show a (y,z)-slice). Both algorithms were executed for 17 cycles.