ANALYSING THE OPTICAL FLOW OF PHOTOMETRIC-STEREO IMAGES

J.R.A. Torreão and B.M. Carvalho

Universidade Federal de Pernambuco, Brazil

INTRODUCTION

As it is well-known, the shading information in two or more images of a surface, obtained under different illuminations from a single camera, can be used for shape estimation in the process known as photometric stereo (PS). In the standard approach to this process, a set of image irradiance equations of the form

$$I_i(s) = R_i(p, q), \quad i \geq 2 \qquad (1)$$

are solved for the surface-gradient components, p and q, at each point $s = (x, y)$ in the image plane – where p and q are given in terms of the surface-height function, $z(x, y)$, as

$$p = \left(\frac{\partial z}{\partial x}\right), \quad q = \left(\frac{\partial z}{\partial y}\right) \qquad (2)$$

In equation (1), the function $R_i(p, q) \equiv R(p, q, \hat{S}_i)$ is the reflectance-map function for the i–th illumination direction $\hat{S}_i$, and $I_i(s)$ is the corresponding image-intensity function [Horn (1), Woodham (2)].

From the observation of the fact that pairs of photometric-stereo images viewed under a stereoscope produce an impression of depth which can be almost as striking as that produced by stereoscopic pairs, we have been led to the study of photometric stereo as a geometric image-matching process. We have thus analysed the possibility of extracting shape information from the optical flow which results from the change of illumination in PS images.

Two main results have arised from our analysis: we have found that, i) under quite general conditions, the photometric-stereo optical flow (henceforth, PS flow) can be related to the spatial derivatives of p and q, and can thus be employed for the estimation of surface curvature; and ii) under the assumption that a linear approximation to the reflectance map is appropriate, estimates of the relative-depth function, $z(x, y)$, can also be obtained from such flow. In what follows, we discuss these results.

Image Processing And Its Applications, 4–6 July 1995
Conference Publication No. 410, © IEE 1995.

PS FLOW AND SURFACE CURVATURE

Let us consider a pair of photometric-stereo images, $I_1(s)$ and $I_2(s)$, corresponding to the illuminations $\hat{S}_1$ and $\hat{S}_2$. If those illumination vectors are not far apart, and if the imaged surface is smooth, we can attempt to match the intensities in the two images, to obtain a disparity field $D(s) = (D_X(s), D_Y(s))$, for which $I_1(s) \approx I_2(s + D(s))$ at each point s in the image plane. Employing a Taylor-series expansion on the right-hand side of the above relation, it is easy to see that the photometric-disparity field satisfies the following constraint:

$$\Delta I(s) \approx D_X(s)\frac{\partial I_2}{\partial x} + D_Y(s)\frac{\partial I_2}{\partial y} \qquad (3)$$

which is the standard optical flow equation, with $\Delta I(s) \equiv I_1(s) - I_2(s)$ playing the role of the time derivative of the image intensity, and with the disparity vector playing the role of the flow velocity [Horn and Schunk (3)].

Instead of dealing with the most general form of the disparity field, let us here consider the disparities arising from a match along a fixed direction in the image plane. For instance, for a match along the X–direction, equation (3) becomes

$$\Delta I(s) \approx D_X(s)\frac{\partial I_2}{\partial x} =$$

$$= D_X(s)\left[\left(\frac{\partial R_2}{\partial p}\right)\left(\frac{\partial p}{\partial x}\right) + \left(\frac{\partial R_2}{\partial q}\right)\left(\frac{\partial q}{\partial x}\right)\right] \qquad (4)$$

with a similar result obtaining for a match along the Y–direction:

$$\Delta I(s) \approx D_Y(s)\frac{\partial I_2}{\partial y} =$$

$$= D_Y(s)\left[\left(\frac{\partial R_2}{\partial p}\right)\left(\frac{\partial p}{\partial y}\right) + \left(\frac{\partial R_2}{\partial q}\right)\left(\frac{\partial q}{\partial y}\right)\right] \qquad (5)$$

If another PS image pair is considered, e.g., $\{I_1, I_3\}$, equations similar to (4) and (5) can be obtained for the X– and Y– matches, involving new disparities,

D'_X and D'_Y, the intensity difference $\Delta I' = I_1 - I_3$, and the reflectance map R_3, of the third image. Given this set of equations, it is straightforward to show that the derivatives of p and q – which correspond to surface curvature – can be estimated from the photometric disparity fields through

$$\mathbf{H} = \begin{pmatrix} \frac{\Delta I}{D_X} & \frac{\Delta I'}{D_{X'}} \\ \frac{\Delta I}{D_Y} & \frac{\Delta I'}{D_{Y'}} \end{pmatrix} \begin{pmatrix} R_{2p} & R_{3p} \\ R_{2q} & R_{3q} \end{pmatrix}^{-1} \tag{6}$$

where $\mathbf{H} = \begin{pmatrix} p_x & q_x \\ p_y & q_y \end{pmatrix}$ is called the Hessian matrix of $z(x, y)$ (The $x-, y-, p-$ and $q-$ subscripts denote differentiation with respect to those variables).

From the Hessian and the gradient of the surface (which can be obtained by the standard PS process), intrinsic object-centered representations of the curvature can be obtained – given for instance in terms of the gaussian and mean curvatures. The relationship between the gaussian curvature and the Hessian, for example, is given by

$$K = \frac{\det \mathbf{H}}{(1 + p^2 + q^2)^2} \tag{7}$$

and thus the sign of the determinant of the Hessian matrix gives the sign of the gaussian curvature, therefore allowing the classification of the surface as elliptic ($K > 0$), hyperbolic ($K < 0$), or parabolic ($K = 0$) [Besl and Jain (4)].

PS FLOW AND RELATIVE DEPTH

Given an image region corresponding to the mean surface orientation (p_0, q_0), a linear approximation to the reflectance-map function can be obtained through a Taylor-series expansion around (p_0, q_0), and equation (1) can then be rewritten as

$$I_i(x, y) \approx k_0^{(i)} + k_1^{(i)}(p - p_0) + k_2^{(i)}(q - q_0) \tag{8}$$

with

$$k_0^{(i)} = R_i(p_0, q_0), \quad k_1^{(i)} = \left(\frac{\partial R_i(p, q)}{\partial p}\right)_0, \quad \text{and}$$

$$k_2^{(i)} = \left(\frac{\partial R_i(p, q)}{\partial q}\right)_0 \tag{9}$$

For a smooth surface, this kind of expansion will give an accurate approximation to the observed intensities around a given point in the image, provided that the neighborhood is chosen small enough so that it contains only a restricted range of (p, q) values [Pentland (5)].

Under the linear approximation, the left-hand side of equation (4) becomes

$$\Delta I(s) \approx k_0 + k_1(p - p_0) + k_2(q - q_0) \approx D_X \frac{\partial I_2}{\partial x} \tag{10}$$

with

$$k_0 = k_0^{(1)} - k_0^{(2)}, \quad k_1 = k_1^{(1)} - k_1^{(2)}, \quad \text{and}$$

$$k_2 = k_2^{(1)} - k_2^{(2)} \tag{11}$$

Quite generally, by choosing the appropriate illumination directions, it is possible to eliminate one of the factors, k_1 or k_2, in (10). For instance, for a lambertian surface illuminated from $\hat{S}_1 = \{\sigma, \tau = \pi\}$, and $\hat{S}_2 = \{\sigma, \tau = 0\}$ (where σ and τ are the slant and tilt angles, respectively), it is easy to show that k_2 is much smaller than k_0 and k_1, when (p_0, q_0) is close to $(0, 0)$. With k_2 eliminated from (10), we thus obtain

$$k_0 + k_1(p - p_0) \approx \frac{\partial}{\partial x}(D_X I_2) - I_2 \frac{\partial D_X}{\partial x} \tag{12}$$

As a first approximation, noting that the disparity field varies slowlier than the image intensities, we can neglect the second term on the right-hand side of equation (12), which can then be manipulated to yield

$$z(x, y) \approx \frac{D_X I_2}{k_1} + \frac{k_1 p_0 - k_0}{k_1} x + F(y) \tag{13}$$

where $F(y)$ is a function only of the $y-$coordinate, and where use was made of equation (2). As a particular instance of (13), for lambertian reflectance and $p_0 = q_0 = 0$, we get

$$z(x, y) \approx \frac{D_X I_2}{2 \sin \sigma} + F(y) \tag{14}$$

It is interesting to remark that the first term on the right-hand side of (14) is exactly the relative depth estimate that would be obtained from a convergent stereoscopic system with vergence angle 2σ, if $D_X I_2$ were the stereoscopic disparity.

The foregoing development can be repeated for a pair of illumination directions such that the $p-$coefficient in the linear expansion of ΔI becomes negligible, when compared to the other two (e.g., for lambertian reflectance and $(p_0, q_0) \approx (0,0)$, with the illuminations $\hat{S}'_1 = \{\sigma, \tau = -\pi/2\}$ and $\hat{S}'_2 = \{\sigma, \tau = \pi/2\}$). In this case, for a match along the Y-direction, we get

$$z(x,y) \approx \frac{D'_Y I'_2}{k'_2} + \frac{k'_2 q_0 - k'_0}{k'_2} y + G(x) \qquad (15)$$

where primed quantities have been employed to emphasize that new illumination directions are being considered.

From the $z(x,y)$ estimates in (13) and (15), we can obtain expressions for the unknown functions $F(y)$ and $G(x)$, up to constant factors. Rewriting relations (13) and (15), for simplicity, as $z(x,y) = A(x,y) + F(y)$ and $z(x,y) = B(x,y) + G(x)$, it is easy to find, by equating the two, that

$$F(y) = \frac{1}{L}\int (B(x,y) - A(x,y))dx \qquad (16)$$

and

$$G(x) = \frac{1}{L}\int (A(x,y) - B(x,y))dy \qquad (17)$$

where the integrations are performed over the image domain, which has been assumed to be of dimension $L \times L$.

Depth estimates can also be obtained which take into account a non-negligible, though small, disparity gradient in equation (12). For a match along the X-direction, for instance, a staightforward manipulation yields

$$z(x,y) \approx$$

$$\approx \frac{D_X(I_2 - k_0^{(2)} + k_1^{(2)} p_0)}{k_1}\left(1 - \frac{k_1^{(2)}}{k_1}\left(\frac{\partial D_X}{\partial x}\right)\right) +$$

$$+\frac{k_1 p_0 - k_0}{k_1}\left(1 - \frac{k_1^{(2)}}{k_1}\left(\frac{\partial D_X}{\partial x}\right)\right) x + H(y) \qquad (18)$$

which, for lambertian reflectance and $p_0 = q_0 = 0$, becomes

$$z(x,y) \approx \frac{D_X(I_2 - \cos\sigma)}{2\sin\sigma}\left(1 - \frac{1}{2}\left(\frac{\partial D_X}{\partial x}\right)\right) +$$

$$+H(y) \qquad (19)$$

EXPERIMENTS

Figures 1 and 2 show results of the application of our approach to the estimation of depth from photometric-stereo images. In each of the experiments, two pairs of such images have been matched through the stochastic algorithm of [Barnard (6)], producing the required photometric-diparity maps.

In the two figures shown, letters (a) and (b) depict one of the input image pairs, while Figure 1(c) and Figure 2(c) represent depth maps which have been recovered via the approximations (14) and (19), respectively.

REFERENCES

1. Horn B.K.P., 1977, "Understanding Image Intensities", Artif. Intell., 8(11), 201-231

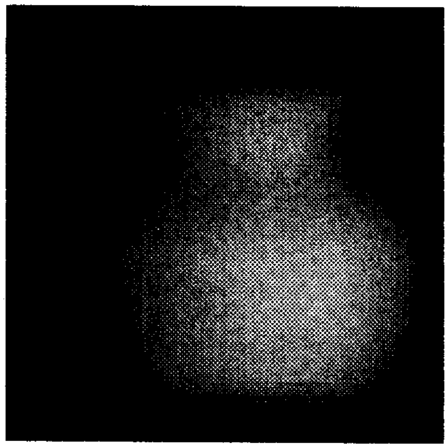
2. Woodham, R.J., 1980 "Photometric Method for Determining Surface Orientation from Multiple Images", Optical Engineering, 19(1), 139-144

3. Horn B.K.P. and Schunck B.G., 1981, "Determining Optical Flow", Artif. Intell., 17, 185-203

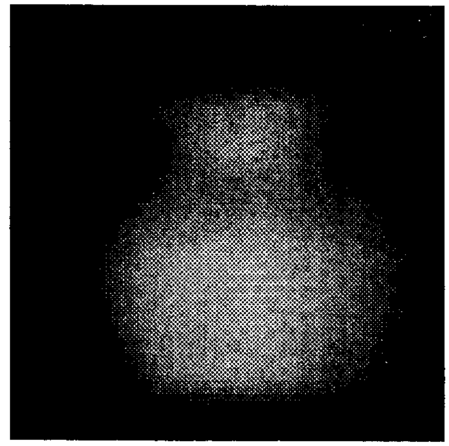
4. Besl P.J. and Jain R., 1985, "Three-Dimensional Object Recognition", Computing Surveys, 17(1), 75-145

5. Pentland, A.P., 1990 "Linear Shape from Shading", Intl. Journal of Computer Vision, 4 153-162

6. Barnard S.T., 1989, "Stochastic Stereo Matching over Scale", Intl. Journal of Computer Vision, 3, 17-32



(a)



(b)

(c)

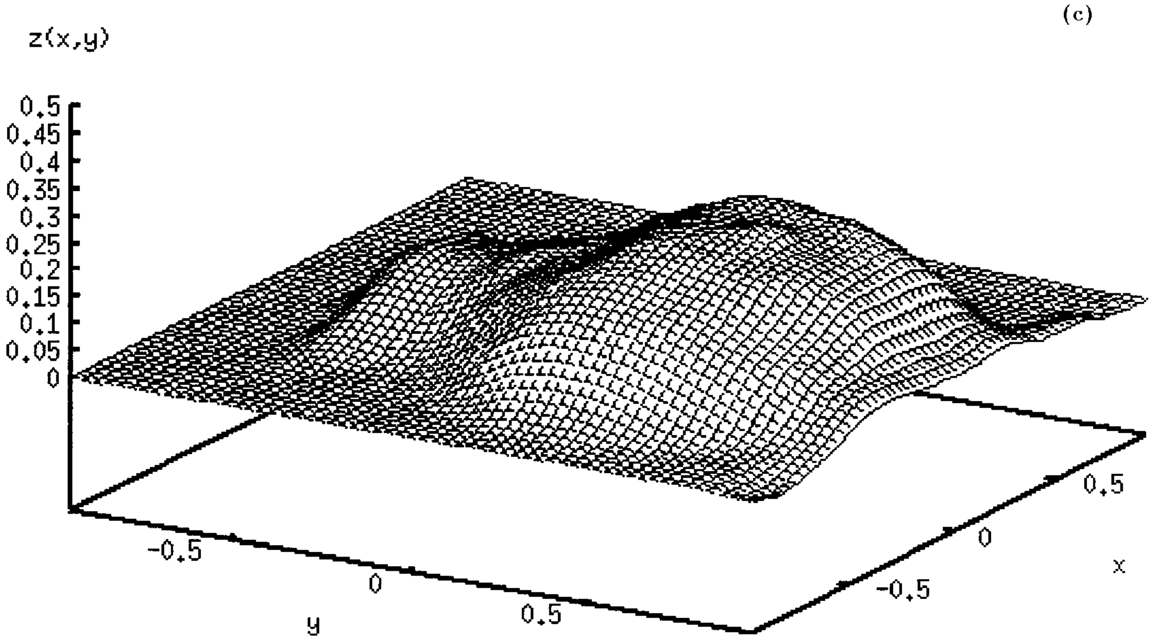
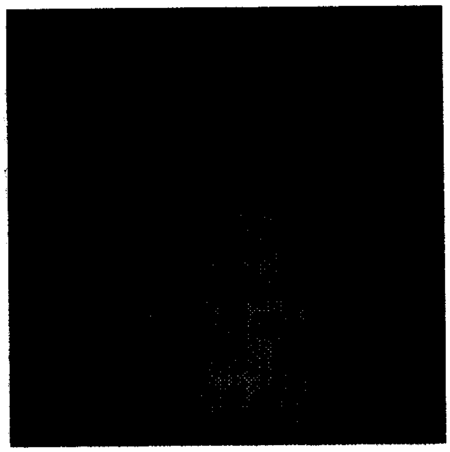
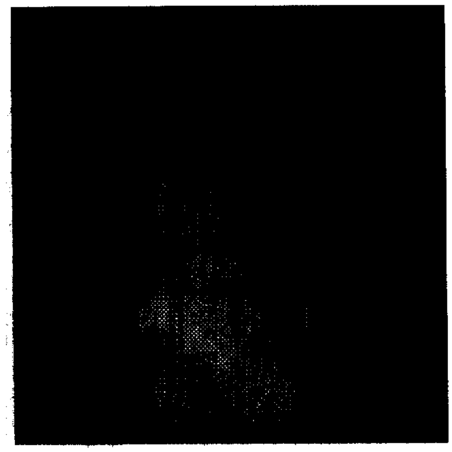


Figure 1 : Depth reconstruction from PS disparities - Equation (14)



(a)



(b)

(c)

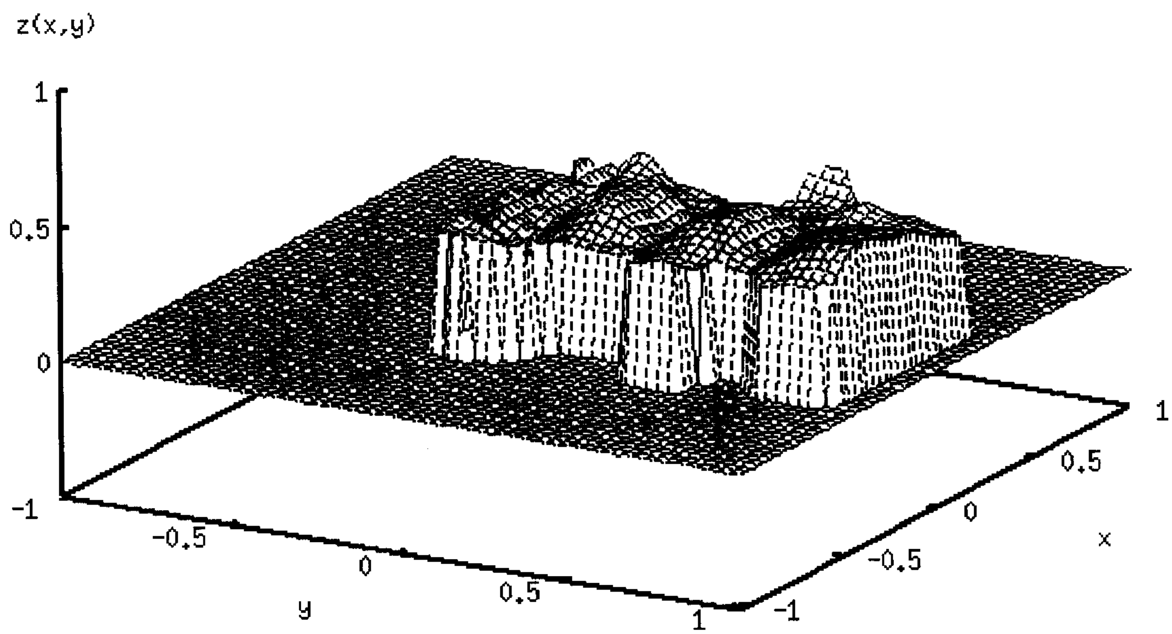


Figure 2 : Depth reconstruction from PS disparities - Equation (19)